The Even & Odd Identities & Verifying Identities

We have already seen the following identities:

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent in Terms of Sine and Cosine:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

The Fundamental Identity:

$$\sin^2 x + \cos^2 x = 1$$

We have also seen the different forms of the Pythagorean Identities and how they are derived from The Fundamental Identity as shown again below.

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

Odd and Even Identities

An odd function is a function for which $f(-x) = -f(x)$, and an even function is one for which $f(-x) = f(x)$. The graph of an odd function is symmetric about the origin, and the graph of an even function is symmetric about the *y*-axis. Of the six trigonometric functions, the cosine and secant are even, and the others are odd.

Odd: $\sin(-x) = -\sin x$ $\qquad \csc(-x) = -\csc x$

$\tan(-x) = -\tan x$ $\qquad \cot(-x) = -\cot x$

Even: $\cos(-x) = \cos x$ $\qquad \sec(-x) = \sec x$

Examples:

$\csc(-x)\tan(-x)$

$\dfrac{1}{1+\cos(-x)}+\dfrac{1}{1-\cos x}$

Verifying Identities with Even and Odd functions

It is often necessary to determine whether two expressions are equivalent to each other. We can use the approaches from the previous section to verify whether equations are identities.

A General Strategy for Verifying Identities

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.
4. Write a single rational expression as a sum of two rational expressions.
5. Combine a sum of two rational expressions into a single rational expression.
6. If both sides simplify to a third expression, then the equation is an identity.

Examples:

Prove that $\dfrac{1-\sin^2 t}{1-\csc(-t)}=\dfrac{1+\sin(-t)}{\csc t}$ is an identity.

Show that $\dfrac{1-\cos^2(-t)}{\sin(-t)}=\tan(-t)\cos(-t)$ is an identit